

A NOTE ON INEXTENSIBLE FLOWS OF CURVES IN E_1^n

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ABSTRACT. In this paper, we study inextensible flows of non-null curves in E_1^n . We give necessary and sufficient conditions for inextensible flow of non-null curve in E_1^n .

1. INTRODUCTION

Flow of curves has a very important place in the field of industry such as modeling ship hulls, buildings, airplane wings, garments, ducts, automobile parts. Moreover Chirikjian and Burdick describe the kinematics of hyperredundant (or "serpentine") robot as the flow of plane curve [2]. The flow of a curve is said to be inextensible if, its arclength is preserved. Firstly, Kwon and Park studied inextensible flows of curves and developable surfaces, which its arclength is preserved, in Euclidean 3-space [10].

Inextensible curve flows conduce to motions in which no strain energy is induced in physical science. For example, the swinging motion of a cord of fixed length can be represented by this type of curve flows. Also inextensible flows of curves have great importance in computer vision and computer animation moreover structural mechanics (see [4],[9], [11]).

There are many studies in the literature on plane curve flows, especially on evolving curves in the direction of their curvature vector field (referred to by various names such as "curve shortening", flow by curvature" and "heat flow"). Among them, perhaps, most important case (but already a very subtle one) is the curve-shortening flow in the plane studied by Gage and Hamilton [5] and Grayson [6]. Another paper about curve flows was studied by Chirikjian [3].

Inextensible flows of curves are studied in many different spaces. For example, Gürbüz have examined inextensible flows of spacelike, timelike and null curves in [7]. After this work Öğrenmiş et al. have studied inextensible curves in Galilean space [12] and Yıldız et al. have studied inextensible flows of curves according to darboux frame in Euclidean 3-space [13] and they have investigated inextensible flows of curves in Euclidean n-space [14], etc.

In the present paper following [10], [7], [12], [13], [14], we study inextensible flows of non-null curves in E_1^n . Further, we give necessary and sufficient conditions for inextensible flows of non-null curves in E_1^n .

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2. PRELIMINARIES AND NOTATIONS

Let E_1^n be the n -dimensional pseudo-Euclidean space with index 1 endowed with the indefinite inner product given by

$$\langle X, Y \rangle = -x_1y_1 + \sum_{i=2}^n x_iy_i,$$

where $X = (x_1, x_2, \dots, x_n)$, $Y = (y_1, y_2, \dots, y_n)$ is the usual coordinate system. An arbitrary vector $X = (x_1, x_2, \dots, x_n)$ in E_1^n can have one of three Lorentzian causal characters; it can be spacelike if $\langle X, X \rangle > 0$ or $X = 0$, timelike if $\langle X, X \rangle < 0$ and null (lightlike) if $\langle X, X \rangle = 0$ and $X \neq 0$. The category into which a given tangent vector falls is called its causal character. These definitions can be generalized for curves as follows. A curve α in E_1^n is said to be spacelike if all of its velocity vectors α' are spacelike, similarly for timelike and null [1].

Let $\alpha : I \subset \mathbb{R} \rightarrow E_1^n$ be non-null curve in E_1^n . A non-null curve $\alpha(s)$ is said to be a unit speed curve if $\langle \alpha'(s), \alpha'(s) \rangle = \varepsilon_0$, (ε_0 being $+1$ or -1 according to α is spacelike or timelike respectively). Let $\{V_1, V_2, \dots, V_n\}$ be the moving Frenet frame along the unit speed curve α , where V_i ($i = 1, 2, \dots, n$) denote i^{th} Frenet vector fields and k_i ($i = 1, 2, \dots, n-1$) denotes the i^{th} curvature function of the curve. Then the Frenet formulas are given as

$$\begin{aligned} V_1' &= k_1 V_2, \\ V_i' &= -\varepsilon_{i-2} \varepsilon_{i-1} k_{i-1} V_{i-1} + k_i V_{i+1}, \quad 1 < i < n, \\ V_n' &= -\varepsilon_{n-2} \varepsilon_{n-1} k_{n-1} V_{n-1} + k_n V_{n+1}, \end{aligned}$$

where $\langle V_i, V_i \rangle = \varepsilon_{i-1} = \mp 1$ [8].

3. INEXTENSIBLE FLOWS OF CURVE IN E_1^n

Unless otherwise stated we assume that.

$$\alpha : [0, l] \times [0, w) \rightarrow E_1^n$$

is a one parameter family of smooth non-null curves in E_1^n , where l is the arclength of the initial curve. Suppose that u is the curve parametrization variable, $0 \leq u \leq l$. If the speed non-null curve α is given by $v = \left\| \frac{d\alpha}{du} \right\|$, then the arclength of α is given as a function of u by

$$s(u) = \int_0^u \left\| \frac{\partial \alpha}{\partial u} \right\| du = \int_0^u v du.$$

The operator $\frac{\partial}{\partial s}$ is given by

$$\frac{\partial}{\partial s} = \frac{1}{v} \frac{\partial}{\partial u}. \quad (3.1)$$

In this case; the arclength is as follows $ds = v du$.

Definition 3.1. Let α be a differentiable non-null curve and $\{V_1, V_2, \dots, V_n\}$ be the Frenet frame of α in Euclidean n -space. Any flow of the non-null curve can be expressed as follows

$$\frac{\partial \alpha}{\partial t} = \sum_{i=1}^n f_i V_i.$$

Here, f_i is the i^{th} scalar speed of the non-null curve α .

Let the arclength variation be

$$s(u, t) = \int_0^u v du.$$

In E_1^n , the requirement that the non-null curve not be subject to any elongation or compression can be expressed by the condition

$$\frac{\partial}{\partial t} s(u, t) = \int_0^u \frac{\partial v}{\partial t} du = 0, \quad u \in [0, l]. \quad (3.2)$$

where $u \in [0, l]$.

Definition 3.2. Let α be a non-null curve in E_1^n . A non-null curve evolution $\alpha(u, t)$ and its flow $\frac{\partial \alpha}{\partial t}$ are said to be inextensible if

$$\frac{\partial}{\partial t} \left\| \frac{\partial \alpha}{\partial u} \right\| = 0.$$

Before deriving the necessary and sufficient condition for inelastic non-null curve flow, we need the following lemma.

Lemma 3.3. Let $\{V_1, V_2, \dots, V_n\}$ be the Frenet frame of non-null curve α and $\frac{\partial \alpha}{\partial t} = \sum_{i=1}^n f_i V_i$ be a smooth flow of α in E_1^n . Then we have the following equality:

$$\frac{\partial v}{\partial t} = \varepsilon_0 \frac{\partial f_1}{\partial u} - \varepsilon_1 f_2 v k_1. \quad (3.3)$$

Proof. As $\frac{\partial}{\partial u}$ and $\frac{\partial}{\partial t}$ commute and $v^2 = \left\langle \frac{\partial \alpha}{\partial u}, \frac{\partial \alpha}{\partial u} \right\rangle$, we have

$$\begin{aligned}
2v \frac{\partial v}{\partial t} &= \frac{\partial}{\partial t} \left\langle \frac{\partial \alpha}{\partial u}, \frac{\partial \alpha}{\partial u} \right\rangle \\
&= 2 \left\langle \frac{\partial \alpha}{\partial u}, \frac{\partial}{\partial t} \left(\sum_{i=1}^n f_i V_i \right) \right\rangle \\
&= 2 \left\langle v V_1, \sum_{i=1}^n \frac{\partial f_i}{\partial u} V_i + \sum_{i=1}^n f_i \frac{\partial V_i}{\partial u} \right\rangle \\
&= 2 \left\langle v V_1, \frac{\partial f_1}{\partial u} V_1 + f_1 \frac{\partial V_1}{\partial u} + \dots + \frac{\partial f_n}{\partial u} V_n + f_n \frac{\partial V_n}{\partial u} \right\rangle \\
&= 2 \left\langle v V_1, \frac{\partial f_1}{\partial u} V_1 + f_1 v k_1 V_2 + \dots + \frac{\partial f_n}{\partial u} V_n - f_n \varepsilon_{n-2} \varepsilon_{n-1} v k_{n-1} V_{n-1} \right\rangle \\
&= 2 \left(\varepsilon_0 \frac{\partial f_1}{\partial u} - \varepsilon_1 f_2 v k_1 \right).
\end{aligned}$$

This clearly forces

$$\frac{\partial v}{\partial t} = \varepsilon_0 \frac{\partial f_1}{\partial u} - \varepsilon_1 f_2 v k_1.$$

□

Theorem 3.4. *Let $\{V_1, V_2, \dots, V_n\}$ be the moving Frenet frame of the non-null curve α and $\frac{\partial \alpha}{\partial t} = \sum_{i=1}^n f_i V_i$ be a differentiable flow of α in E_1^n . In this case, the flow is inextensible if and only if*

$$\frac{\partial f_1}{\partial s} = \varepsilon_0 \varepsilon_1 f_2 k_1. \quad (3.4)$$

Proof. Let us assume that the non-null curve flow is inextensible. From equations (3.2) and (3.3) it follows that

$$\frac{\partial}{\partial t} s(u, t) = \int_0^u \frac{\partial v}{\partial t} du = \int_0^u \left(\varepsilon_0 \frac{\partial f_1}{\partial u} - \varepsilon_1 f_2 v k_1 \right) du = 0, \quad u \in [0, l].$$

This clearly forces

$$\varepsilon_0 \frac{\partial f_1}{\partial u} - \varepsilon_1 f_2 v k_1 = 0.$$

Combining the last equation with (3.1) yields

$$\frac{\partial f_1}{\partial s} = \varepsilon_0 \varepsilon_1 f_2 k_1.$$

On the contrary, following similar way as above, the proof can be completed.

Now, suppose that the non-null curve α is a arclength parametrized curve. That is, $v = 1$ and the local coordinate u corresponds to the curve arclength s .

Lemma 3.5. *Let $\{V_1, V_2, \dots, V_n\}$ be the moving Frenet frame of the non-null curve α . The differentiations of $\{V_1, V_2, \dots, V_n\}$ with respect to t is*

$$\begin{aligned}\frac{\partial V_1}{\partial t} &= \left[\sum_{i=2}^{n-1} \left(f_{i-1} k_{i-1} + \frac{\partial f_i}{\partial s} - \varepsilon_{i-1} \varepsilon_i f_{i+1} k_i \right) V_i \right] + \left(f_{n-1} k_{n-1} + \frac{\partial f_n}{\partial s} \right) V_n, \\ \frac{\partial V_j}{\partial t} &= -\varepsilon_0 \left(\varepsilon_{j-1} f_{j-1} k_{j-1} + \varepsilon_{j-1} \frac{\partial f_j}{\partial s} - \varepsilon_j f_{j+1} k_j \right) V_1 + \left[\sum_{\substack{k=2 \\ k \neq j}}^n \Psi_{kj} V_k \right], \quad 1 < j < n, \\ \frac{\partial V_n}{\partial t} &= -\varepsilon_0 \varepsilon_{n-1} \left(f_{n-1} k_{n-1} + \frac{\partial f_n}{\partial s} \right) V_1 + \left[\sum_{k=2}^{n-1} \Psi_{kn} V_k \right],\end{aligned}$$

where $\Psi_{kj} = \left\langle \frac{\partial V_j}{\partial t}, V_k \right\rangle$, $k \neq j$, $1 \leq j, k \leq n$ and $\varepsilon_{i-1} = \langle V_i, V_i \rangle = \pm 1$, $1 \leq i \leq n$.

Proof. As $\frac{\partial}{\partial t}$ and $\frac{\partial}{\partial s}$ commute, we have

$$\begin{aligned}\frac{\partial V_1}{\partial t} &= \frac{\partial}{\partial t} \left(\frac{\partial \alpha}{\partial s} \right) = \frac{\partial}{\partial s} \left(\frac{\partial \alpha}{\partial t} \right) = \frac{\partial}{\partial s} \left(\sum_{i=1}^n f_i V_i \right) = \sum_{i=1}^n \frac{\partial f_i}{\partial s} V_i + \sum_{i=1}^n f_i \frac{\partial V_i}{\partial s} \\ &= \frac{\partial f_1}{\partial s} V_1 + f_1 \frac{\partial V_1}{\partial s} + \frac{\partial f_2}{\partial s} V_2 + f_2 \frac{\partial V_2}{\partial s} + \dots + \frac{\partial f_n}{\partial s} V_n + f_n \frac{\partial V_n}{\partial s} \\ &= \frac{\partial f_1}{\partial s} V_1 + f_1 k_1 V_2 + \frac{\partial f_2}{\partial s} V_2 + f_2 (-\varepsilon_0 \varepsilon_1 k_1 V_1 + k_2 V_3) + \dots + \frac{\partial f_n}{\partial s} V_n - f_n \varepsilon_{n-2} \varepsilon_{n-1} k_{n-1} V_{n-1}.\end{aligned}$$

Substituting the equation (3.4) into the last equation yields

$$\frac{\partial V_1}{\partial t} = \left[\sum_{i=2}^{n-1} \left(f_{i-1} k_{i-1} + \frac{\partial f_i}{\partial s} - \varepsilon_{i-1} \varepsilon_i f_{i+1} k_i \right) V_i \right] + \left(f_{n-1} k_{n-1} + \frac{\partial f_n}{\partial s} \right) V_n.$$

Now, differentiating the Frenet frame with respect to t for $1 < j < n$

$$\begin{aligned}0 &= \frac{\partial}{\partial t} \langle V_1, V_j \rangle = \left\langle \frac{\partial V_1}{\partial t}, V_j \right\rangle + \left\langle V_1, \frac{\partial V_j}{\partial t} \right\rangle \\ &= \left(\varepsilon_{j-1} f_{j-1} k_{j-1} + \varepsilon_{j-1} \frac{\partial f_j}{\partial s} - \varepsilon_j f_{j+1} k_j \right) + \left\langle V_1, \frac{\partial V_j}{\partial t} \right\rangle.\end{aligned}\tag{3.5}$$

Thus, from the last equation we get

$$\frac{\partial V_j}{\partial t} = -\varepsilon_0 \left(\varepsilon_{j-1} f_{j-1} k_{j-1} + \varepsilon_{j-1} \frac{\partial f_j}{\partial s} - \varepsilon_j f_{j+1} k_j \right) V_1 + \left[\sum_{\substack{k=2 \\ k \neq j}}^n \Psi_{kj} V_k \right].$$

Since $\langle V_1, V_n \rangle = 0$, this follows by the similar method as above

$$\frac{\partial V_n}{\partial t} = -\varepsilon_0 \varepsilon_{n-1} \left(f_{n-1} k_{n-1} + \frac{\partial f_n}{\partial s} \right) V_1 + \left[\sum_{k=2}^{n-1} \Psi_{kn} V_k \right].$$

Theorem 3.6. *Let the non-null curve flow $\frac{\partial \alpha}{\partial t} = \sum_{i=1}^n f_i V_i$ be inextensible in E_1^n .*

Then, there exist the following system of partial differential equations.

$$\begin{aligned}\frac{\partial k_1}{\partial t} &= \varepsilon_0 \varepsilon_1 f_2 k_1^2 + f_1 \frac{\partial k_1}{\partial s} + \frac{\partial^2 f_2}{\partial s^2} - 2\varepsilon_1 \varepsilon_2 \frac{\partial f_3}{\partial s} k_2 - \varepsilon_1 \varepsilon_2 f_3 \frac{\partial k_2}{\partial s} - \varepsilon_1 \varepsilon_2 f_2 k_2^2 - \varepsilon_1 \varepsilon_3 f_4 k_2 k_3, \\ \frac{\partial k_{i-1}}{\partial t} &= -\varepsilon_{i-2} \varepsilon_{i-1} \frac{\partial \Psi_{(i-1)i}}{\partial s} - \varepsilon_{i-2} \varepsilon_{i-1} \Psi_{(i-2)i} k_{i-2}, \\ \frac{\partial k_i}{\partial t} &= \frac{\partial \Psi_{(i+1)i}}{\partial s} - \varepsilon_i \varepsilon_{i+1} \Psi_{(i+2)i} k_{i+1}, \\ \frac{\partial k_{n-1}}{\partial t} &= -\varepsilon_{n-2} \varepsilon_{n-1} \frac{\partial \Psi_{(n-1)n}}{\partial s} - \varepsilon_{n-2} \varepsilon_{n-1} \Psi_{(n-2)n} k_{n-2}.\end{aligned}$$

□

□

Proof. Noting that $\frac{\partial}{\partial s} \frac{\partial V_1}{\partial t} = \frac{\partial}{\partial t} \frac{\partial V_1}{\partial s}$ thus we have

$$\begin{aligned}\frac{\partial}{\partial s} \frac{\partial V_1}{\partial t} &= \frac{\partial}{\partial s} \left[\sum_{i=2}^{n-1} \left(f_{i-1} k_{i-1} + \frac{\partial f_i}{\partial s} - \varepsilon_{i-1} \varepsilon_i f_{i+1} k_i \right) V_i + \left(f_{n-1} k_{n-1} + \frac{\partial f_n}{\partial s} \right) V_n \right] \\ &= \sum_{i=2}^{n-1} \left[\left(\frac{\partial f_{i-1}}{\partial s} k_{i-1} + f_{i-1} \frac{\partial k_{i-1}}{\partial s} + \frac{\partial^2 f_i}{\partial s^2} - \varepsilon_{i-1} \varepsilon_i \frac{\partial f_{i+1}}{\partial s} k_i - \varepsilon_{i-1} \varepsilon_i f_{i+1} \frac{\partial k_i}{\partial s} \right) V_i \right] \\ &\quad + \sum_{i=2}^{n-1} \left[\left(f_{i-1} k_{i-1} + \frac{\partial f_i}{\partial s} - \varepsilon_{i-1} \varepsilon_i f_{i+1} k_i \right) \frac{\partial V_i}{\partial s} \right] \\ &\quad + \left(\frac{\partial f_{n-1}}{\partial s} k_{n-1} + f_{n-1} \frac{\partial k_{n-1}}{\partial s} + \frac{\partial^2 f_n}{\partial s^2} \right) V_n + \left(f_{n-1} k_{n-1} + \frac{\partial f_n}{\partial s} \right) \frac{\partial V_n}{\partial s}\end{aligned}\tag{3.6}$$

while

$$\frac{\partial}{\partial t} \frac{\partial V_1}{\partial s} = \frac{\partial}{\partial t} (k_1 V_2) = \frac{\partial k_1}{\partial t} V_2 + k_1 \frac{\partial V_2}{\partial t}.\tag{3.7}$$

Therefore, from the equation (3.6) and (3.7) it is seen that

$$\frac{\partial k_1}{\partial t} = \varepsilon_0 \varepsilon_1 f_2 k_1^2 + f_1 \frac{\partial k_1}{\partial s} + \frac{\partial^2 f_2}{\partial s^2} - 2\varepsilon_1 \varepsilon_2 \frac{\partial f_3}{\partial s} k_2 - \varepsilon_1 \varepsilon_2 f_3 \frac{\partial k_2}{\partial s} - \varepsilon_1 \varepsilon_2 f_2 k_2^2 - \varepsilon_1 \varepsilon_3 f_4 k_2 k_3.$$

Since $\frac{\partial}{\partial s} \frac{\partial V_i}{\partial t} = \frac{\partial}{\partial t} \frac{\partial V_i}{\partial s}$, we obtain

$$\begin{aligned}\frac{\partial}{\partial s} \frac{\partial V_i}{\partial t} &= \frac{\partial}{\partial s} \left[-\varepsilon_0 \left(\varepsilon_{i-1} f_{i-1} k_{i-1} + \varepsilon_{i-1} \frac{\partial f_i}{\partial s} - \varepsilon_i f_{i+1} k_i \right) V_1 + \left(\sum_{\substack{k=2 \\ k \neq i}}^n \Psi_{ki} V_k \right) \right] \\ &= \varepsilon_0 \left(-\varepsilon_{i-1} \frac{\partial f_{i-1}}{\partial s} k_{i-1} - \varepsilon_{i-1} f_{i-1} \frac{\partial k_{i-1}}{\partial s} - \varepsilon_{i-1} \frac{\partial^2 f_i}{\partial s^2} + \varepsilon_i \frac{\partial f_{i+1}}{\partial s} k_i + \varepsilon_i f_{i+1} \frac{\partial k_i}{\partial s} \right) V_1 \\ &\quad + \left(-\varepsilon_0 \varepsilon_{i-1} f_{i-1} k_{i-1} - \varepsilon_0 \varepsilon_{i-1} \frac{\partial f_i}{\partial s} + \varepsilon_0 \varepsilon_i f_{i+1} k_i \right) \frac{\partial V_1}{\partial s} + \sum_{\substack{k=2 \\ k \neq i}}^n \left(\frac{\partial \Psi_{ki}}{\partial s} V_k + \Psi_{ki} \frac{\partial V_k}{\partial s} \right)\end{aligned}$$

while

$$\begin{aligned}\frac{\partial}{\partial t} \frac{\partial V_i}{\partial s} &= \frac{\partial}{\partial t} (-\varepsilon_{i-2}\varepsilon_{i-1}k_{i-1}V_{i-1} + k_iV_{i+1}) \\ &= -\varepsilon_{i-2}\varepsilon_{i-1}\frac{\partial k_{i-1}}{\partial t}V_{i-1} - \varepsilon_{i-2}\varepsilon_{i-1}k_{i-1}\frac{\partial V_{i-1}}{\partial t} + \frac{\partial k_i}{\partial t}V_{i+1} + k_i\frac{\partial V_{i+1}}{\partial t}.\end{aligned}$$

Hence

$$\frac{\partial k_{i-1}}{\partial t} = -\varepsilon_{i-2}\varepsilon_{i-1}\frac{\partial \Psi_{(i-1)i}}{\partial s} - \varepsilon_{i-2}\varepsilon_{i-1}\Psi_{(i-2)i}k_{i-2}$$

and

$$\frac{\partial k_i}{\partial t} = \frac{\partial \Psi_{(i+1)i}}{\partial s} - \varepsilon_i\varepsilon_{i+1}\Psi_{(i+2)i}k_{i+1}.$$

By same way as above and considering $\frac{\partial}{\partial s} \frac{\partial V_n}{\partial t} = \frac{\partial}{\partial t} \frac{\partial V_n}{\partial s}$ we reach

$$\frac{\partial k_{n-1}}{\partial t} = -\varepsilon_{n-2}\varepsilon_{n-1}\frac{\partial \Psi_{(n-1)n}}{\partial s} - \varepsilon_{n-2}\varepsilon_{n-1}\Psi_{(n-2)n}k_{n-2}.$$

□

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